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AN ORDER STATISTIC APPROACH TO FUZE DESIGN

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April 1981

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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20. ABSTRACT - Contd.

in a fuzing system. A computer program has been written which provides the probability of a warhead fuzing as a function of the parameters which characterize the detonators. Conversely, given a required probability of fuzing, the program will determine the necessary detonator characteristics. Although motivated by this specific problem, the work is general in nature and should have additional applications in the armament research and development community.

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I. INTRODUCTION

Let X_1, X_2, \dots, X_N be independent, identically-distributed random variables. Then $Y_1 \leq Y_2 \leq \dots \leq Y_N$, where the Y_i 's are the X_i 's rearranged in order of increasing magnitudes, are defined to be the order statistics corresponding to the original random sample. Order statistics find immediate application in the design and evaluation of logical structures which make decisions based on the relative values assumed by a set of n random variables. One particularly interesting application involves the determination of time windows for firing impulses in a fuzing system. This is the problem which motivated the work on which we are reporting. However, the work is general in nature and may prove useful for other applications in the armament research and development community.

II. STATEMENT OF THE PROBLEM

In the particular problem which we addressed, a fuze contains N detonators, K of which must function within a specific time span. Furthermore, the second detonator (which functions at time Y_2) partitions the time span into two sub-intervals. The first subinterval $[Y_2 - \delta_1, Y_2]$ is examined to determine if the first detonator functioned within that time segment, and the second subinterval $[Y_2, Y_2 + \delta_2]$ is monitored to count the number of additional detonators activated during that period of time. If, within the time interval $[Y_2 - \delta_1, Y_2 + \delta_2]$, K detonators have functioned, then the command to fire will be initiated; otherwise, it will not. The times to function for the detonators are random variables and, as such, can be characterized by a cumulative distribution function F . Assuming that the time to function of each detonator is identically distributed, then the problem consists of expressing the probability of fuzing as a function of K, N, δ_1, δ_2 , and F .

III. SOLUTION

Let X_i be the time to function of detonator i in its operating environment. Then X_1, X_2, \dots, X_N are independent, identically-distributed random variables; and we can define Y_1, Y_2, \dots, Y_N to be the order statistics corresponding to the X_i 's. For our problem, if $Y_2 - Y_1 \leq \delta_1$, we are interested in the probability that $Y_K - Y_2 \leq \delta_2$; however, if $Y_2 - Y_1 > \delta_1$, we need to determine the probability that $Y_{K+1} - Y_2 \leq \delta_2$ assuming $K + 1 \leq N$. Therefore, we need to evaluate

$$\begin{aligned} \text{Pr \{warhead fuzing\}} &= \text{Pr \{ } Y_2 - Y_1 \leq \delta_1 \} \text{Pr \{ } Y_K - Y_2 \leq \delta_2 \mid Y_2 - Y_1 \leq \delta_1 \}} \\ &+ \text{Pr \{ } Y_2 - Y_1 > \delta_1 \} \text{Pr \{ } Y_{K+1} - Y_2 \leq \delta_2 \mid Y_2 - Y_1 > \delta_1 \}}. \end{aligned} \quad (1)$$

Applying the definition of conditional probability we obtain

$$\begin{aligned} \Pr \{\text{warhead fuzing}\} = & \Pr \{Y_2 - Y_1 \leq \delta_1\} \frac{\Pr \{Y_K - Y_2 \leq \delta_2 \text{ and } Y_2 - Y_1 \leq \delta_1\}}{\Pr \{Y_2 - Y_1 \leq \delta_1\}} \\ & + \Pr \{Y_2 - Y_1 > \delta_1\} \frac{\Pr \{Y_{K+1} - Y_2 \leq \delta_2 \text{ and } Y_2 - Y_1 > \delta_1\}}{\Pr \{Y_2 - Y_1 > \delta_1\}}, \quad (2) \end{aligned}$$

which upon simplifying yields

$$\begin{aligned} \Pr \{\text{warhead fuzing}\} = & \Pr \{Y_K - Y_2 \leq \delta_2 \text{ and } Y_2 - Y_1 \leq \delta_1\} \\ & + \Pr \{Y_{K+1} - Y_2 \leq \delta_2 \text{ and } Y_2 - Y_1 > \delta_1\}. \quad (3) \end{aligned}$$

Defining

$$F(a) = \int_{-\infty}^a f(x) dx, \quad (4)$$

we can proceed to evaluate the first term on the right-hand side of Equation 3. As shown in Appendix A we can obtain the joint probability density function of the 1st, 2nd, and Kth order statistics,

$$\begin{aligned} f(y_1, y_2, y_K) = & \frac{N!}{(K-3)!(N-K)!} f(y_1) f(y_2) [F(y_K) - F(y_2)]^{K-3} \\ & \cdot f(y_K) [1 - F(y_K)]^{N-K} \quad (5) \end{aligned}$$

If we let $u = y_K - y_2$, $v = y_2 - y_1$, and $w = y_1$, then we can rewrite Equation 5,

$$\begin{aligned} f(u, v, w) = & \frac{N!}{(K-3)!(N-K)!} f(w) f(v+w) [F(u+v+w) - F(v+w)]^{K-3} \\ & \cdot f(u+v+w) [1 - F(u+v+w)]^{N-K}, \quad (6) \end{aligned}$$

and the desired probability is

$$\int_{-\infty}^{+\infty} \int_0^{\delta_1} \int_0^{\delta_2} f(u, v, w) du dv dw \quad (7)$$

which is equal to

$$J_1 = \frac{N!}{(K-3)!(N-K)!} \int_{-\infty}^{+\infty} f(w) \int_0^{\delta_1} f(v+w) \int_0^{\delta_2} [F(u+v+w) - F(v+w)]^{K-3} \cdot f(u+v+w) [1 - F(u+v+w)]^{N-K} du dv dw \quad (8)$$

In an analogous manner we can obtain the joint probability density function of the 1st, 2nd, and K+1st order statistics; and, letting $u = y_{K+1} - y_2$, $v = y_2 - y_1$ and $w = y_1$, we can obtain the necessary probability for the second term on the right-hand side of Equation 3. That probability is equal to

$$J_2 = \frac{N!}{(K-2)!(N-K-1)!} \int_{-\infty}^{+\infty} f(w) \int_{\delta_1}^{+\infty} f(v+w) \int_0^{\delta_2} [F(u+v+w) - F(v+w)]^{K-2} \cdot f(u+v+w) [1 - F(u+v+w)]^{N-K-1} du dv dw \quad (9)$$

The probability of fuzing is then just the sum of Equation 8 and Equation 9.

Appendix B contains a computer program which evaluates these integrals. In its current form it assumes that the distribution of the times to function of the detonators is normal; however, it can be easily modified to change this distribution. The program requires as input N , K , δ_1 , δ_2 , and σ (standard deviation of the assumed distribution).

IV. RESULTS

For the problem we addressed, the value for σ was specified to be 10^{-5} seconds. Figure 1 presents the results of changing δ_1 and δ_2 for a fuzing system in which K is equal to $N-1$. In Figure 2 we considered a fuze with eight detonators; and, keeping σ equal to 10^{-5} , we varied K as well as δ_1 and δ_2 . Of course, given any four of the five input variables (σ , K , N , δ_1 , and δ_2), we can obtain a specific probability of warhead fuzing by parametrically varying the remaining variable.

V. APPLICATION TO FUZE DESIGN

An important consideration is to determine optimal δ_1 and δ_2 based on a specified σ , K , and N and a desired probability of warhead fuzing. To do this several factors must be considered. To maintain the reliability of the fuze it is necessary that the interval $[Y_2 - \delta_1, Y_2 + \delta_2]$ be sufficiently

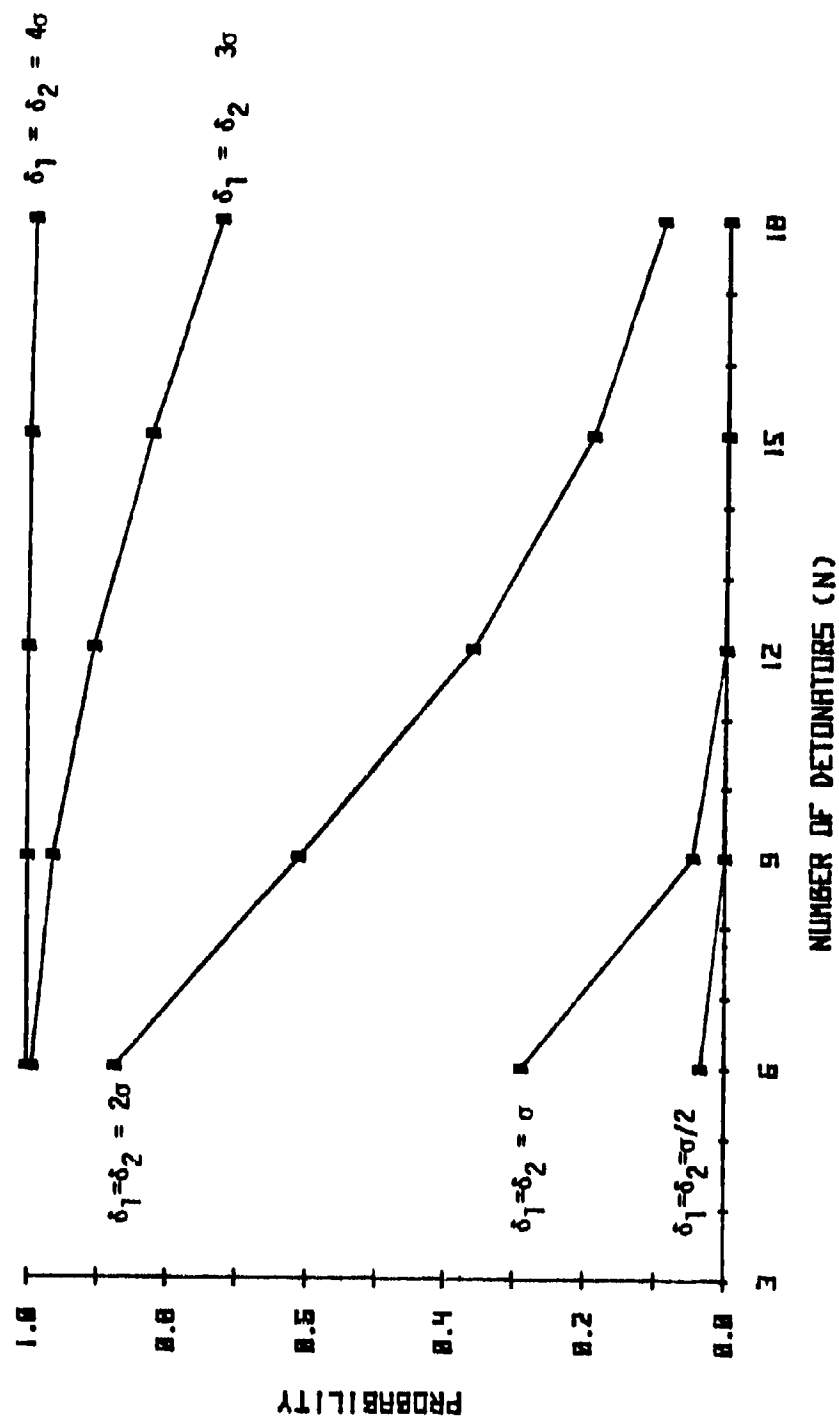


FIGURE 1. PROBABILITY OF WARHEAD FUZING IN AN N-1 OUT OF N FUZING SYSTEM ($\sigma = .00001$).

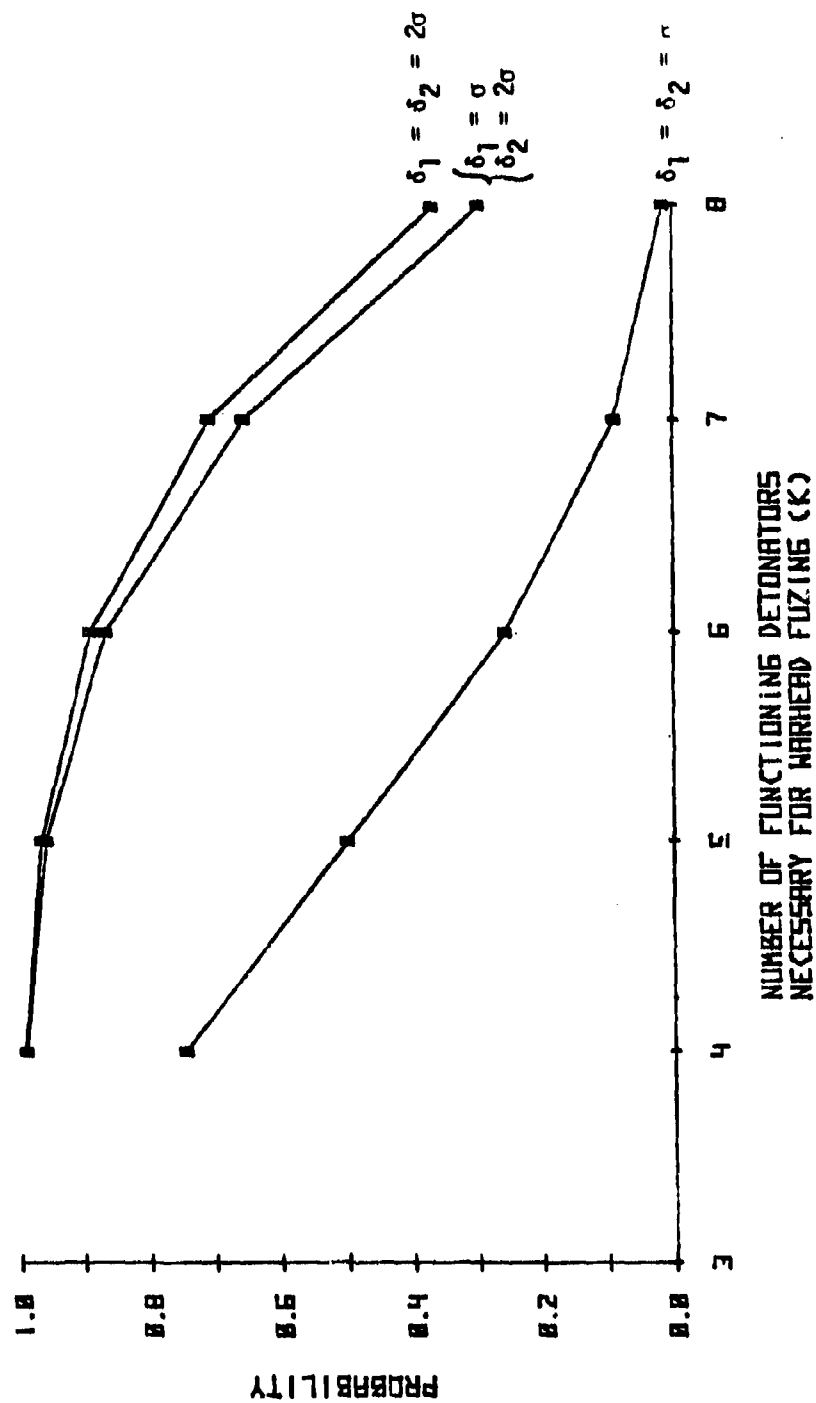


FIGURE 2. PROBABILITY OF WARHEAD FUZING IN A K OUT OF 8 FUZING SYSTEM ($\sigma = .00001$).

large to allow for the occurrence of K detonations. However, safety must also be considered; and here it is desirable that the interval $[Y_2 - \delta_1, Y_2 + \delta_2]$ be as small as possible to preclude the occurrence of spurious detonations. These conflicting goals could be accomplished through a solution to the following optimization problem:

Given σ , K, and N, determine $\delta_1, \delta_2 > 0$ for which $\delta_1 + \delta_2$ is minimum subject to $J_1 + J_2 > \alpha$ (reference Equation 8 and Equation 9) where α is the desired probability of warhead fuzing.

Consideration of this problem is beyond the scope of this report. Here, we will content ourselves with the determination of reasonable initial values of δ_1 and δ_2 for use in the computer algorithm. Such values may then be adjusted as required to obtain the desired probability of warhead fuzing. For this purpose practical initial values for δ_1 and δ_2 would be $E[Y_2 - Y_1]$ and $E[Y_K - Y_2]$ respectively, where $E[.]$ is the expected value of a random variable.

For the general case we can define $W_{RS} = Y_S - Y_R$, and from consideration of the material in Appendix A we obtain the probability density function

$$f(w_{RS}) = f(y_S - y_R) = C_{RS} \int_{-\infty}^{\infty} F(x)^{R-1} f(x) \cdot [F(x + w_{RS}) - F(x)]^{S-R-1} f(x + w_{RS}) \cdot [1 - F(x + w_{RS})]^{N-S} dx \quad (10)$$

$$\text{where } C_{RS} = \frac{N!}{(R-1)! (S-R-1)! (N-S)!}.$$

To determine $E[W_{RS}] = E[Y_S - Y_R]$ we must evaluate

$$E[W_{RS}] = \int_0^{+\infty} w_{RS} f(w_{RS}) dw_{RS}. \quad (11)$$

Therefore, we can obtain $E[Y_2 - Y_1]$ by evaluating

$$E[W_{12}] = C_{12} \int_0^{+\infty} \int_{-\infty}^{+\infty} w_{12} f(x) f(x + w_{12}) \cdot [1 - F(x + w_{12})]^{N-2} dx dw_{12}; \quad (12)$$

and in an analogous manner we can obtain $E[Y_K - Y_2]$ by evaluating

$$E[W_{2K}] = C_{2K} \int_0^{+\infty} \int_{-\infty}^{+\infty} w_{2K} F(x) f(x) \cdot [F(x + w_{2K}) - F(x)]^{K-3} f(x + w_{2K}) \cdot [1 - F(x + w_{2K})]^{N-K} dx dw_{2K} . \quad (13)$$

The p th quasi-range of N sample values is defined to be the range of $(N - 2p)$ values, omitting the p largest and the p smallest. For the special case in which $K = N-1$, $W_{2K} = Y_{N-1} - Y_2$ becomes the first quasi-range; and this distribution has been investigated and, to some extent, tabulated. For example, H. Leon Harter* has provided the expected values of the first quasi-range = $E[W_{2, N-1}]$ for samples from a normal distribution with mean zero and variance one and for values of N from four to one hundred.

ACKNOWLEDGEMENTS

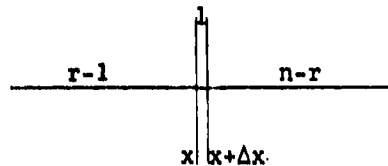
We would like to acknowledge Professor H. A. David of Iowa State University for his personal correspondence directed toward the evaluation of the joint distribution of order statistics; also, Mr. Denis Silvia of the Ballistic Research Laboratory for providing the problem which prompted this work.

* Harter, H. L., "The Use of Sample Quasi-Ranges in Estimating Population Standard Deviation," The Annals of Mathematical Statistics, Vol. 30, No. 4, 1959, Pp. 980-999.

APPENDIX A

DERIVATION OF THE JOINT PROBABILITY DENSITY FUNCTION

As shown by H. A. David,* we can obtain the joint probability density function for Y_1, Y_2, \dots, Y_n . Let X_1, X_2, \dots, X_n be a random sample from a population with density $f(x) = F'(x)$; and let Y_1, Y_2, \dots, Y_n be the corresponding order statistics. Then the probability density function for the r th order statistic may be derived by considering the following configuration:



That is, $X_1 \leq x$ for $r-1$ of the X_i , $x < X_1 \leq x + \Delta x$ for one X_i , and $X_1 > x + \Delta x$ for the remaining $n-r$ of the X_i . The number of ways this combination of events can occur is

$$\frac{n!}{(r-1)! 1! (n-r)!},$$

and each such way has probability

$$[F(x)]^{r-1} [F(x+\Delta x) - F(x)]^1 [1 - F(x+\Delta x)]^{n-r}.$$

Therefore, we have

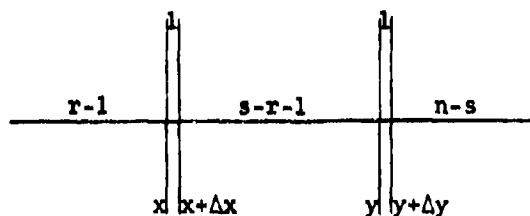
$$\begin{aligned} \Pr \{x < Y_r \leq x + \Delta x\} &= \frac{n!}{(r-1)! (n-r)!} \\ &\cdot [F(x)]^{r-1} [F(x+\Delta x) - F(x)] [1 - F(x+\Delta x)]^{n-r} + O(\Delta x^2) \end{aligned} \quad (A1)$$

where $O(\Delta x^2)$ means terms of order $(\Delta x)^2$ and includes the probability of realizations of $x < Y_r \leq x + \Delta x$ in which more than one X_i is in $(x, x + \Delta x)$. Dividing both sides of Equation A1 by Δx and then letting $\Delta x \rightarrow 0$, we obtain

$$f_{Y_r}(x) = \frac{n!}{(r-1)! (n-r)!} [F(x)]^{r-1} f(x) [1-F(x)]^{n-r}. \quad (A2)$$

*David, H. A., Order Statistics, John Wiley and Sons, New York, 1970, Pp. 9-10.

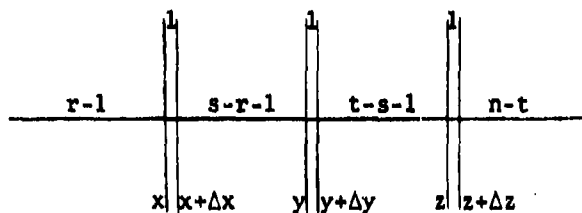
In a similar manner we can derive the joint probability density function of Y_r and Y_s :



and through an analogous argument we obtain

$$f_{Y_r, Y_s}(x, y) = \frac{n!}{(r-1)!(s-r-1)!(n-s)!} [F(x)]^{r-1} f(x) \cdot [F(y) - F(x)]^{s-r-1} f(y) [1 - F(y)]^{n-s}. \quad (A3)$$

Finally, for the joint probability density function of Y_r , Y_s , and Y_t :



and

$$f_{Y_r, Y_s, Y_t}(x, y, z) = \frac{n!}{(r-1)!(s-r-1)!(t-s-1)!(n-t)!} [F(x)]^{r-1} f(x) \cdot [F(y) - F(x)]^{s-r-1} f(y) [F(z) - F(y)]^{t-s-1} f(z) [1 - F(z)]^{n-t}. \quad (A4)$$

For the case $r=1$, $s=2$, and $t=k$, we obtain

$$f_{Y_1, Y_2, Y_k}(x, y, z) = \frac{n!}{(k-3)!(n-k)!} f(x) f(y) \cdot [F(z) - F(y)]^{k-3} f(z) [1 - F(z)]^{n-k}. \quad (A5)$$

APPENDIX B

COMPUTER PROGRAM

We are presenting here the computer program which evaluates the desired probability function. As noted in the program comments, we have assumed that the times to function of the detonators are normally distributed with mean zero and variance σ^2 . However, with just a few changes to the subroutines, a different distribution may be assumed. To do this, all cards containing "NORMAL" in columns 74 through 79 must be replaced by others with the appropriate probability density function or cumulative distribution function.


```

PROGRAM DETON (INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)
-----

```

```

      THIS PROGRAM DETERMINES THE PROBABILITY THAT K OUT OF N
      DETONATORS WILL FUNCTION WITHIN A GIVEN TIMESPAN. K MUST
      BE GREATER THAN OR EQUAL TO 3, AND K MUST BE LESS THAN OR
      EQUAL TO N. THE DISTRIBUTION OF THEIR FUNCTIONING IS ASSUMED
      TO BE GAUSSIAN WITH A MEAN EQUAL TO 0.0 AND A STANDARD
      DEVIATION EQUAL TO SIGMA. THE SOLUTION IS DERIVED THROUGH
      THE USE OF ORDER STATISTICS AND IS OBTAINED BY EVALUATING
      A TRIPLE INTEGRAL.

```

```

      IF THE NUMBER OF DETONATORS IS GREATER THAN THIRTY,
      THEN THE TOLERANCE LIMITS OF THE INTEGRALS MUST BE REDEFINED.
      THAT IS, THE VARIABLE 'ERROR' IN THE MAIN ROUTINE AND THE
      VARIABLES 'TOLX,TOLY' IN SUBROUTINE 'DIST' SHOULD BE ADJUSTED.
      WITH THE TOLERANCE LIMITS CURRENTLY IN THE PROGRAM,
      THE RESULTING PROBABILITIES ARE CORRECT TO TWO DECIMAL PLACES.

```

```

      INPUT IS AS FOLLOWS ....

```

```

      N ..... NUMBER OF DETONATORS
      K ..... NUMBER OF DETONATORS TO FUNCTION
      SIGMA ..... STANDARD DEVIATION OF DISTRIBUTION
      DELT1 ..... TIMESPAN (FIRST TO SECOND DETONATORS)
      DELT2 ..... TIMESPAN (SECOND TO LAST DETONATORS)

```

```

      REAL NFACT,K2FACT,K3FACT,NKFACT,NK1FACT

```

```

      DIMENSION IERR(6)

```

```

      COMMON /COM1/ N,K,SIGMA,DELT1,DELT2
      COMMON /COM2/ PI,W
      COMMON /COM4/ IND

```

```

      EXTERNAL DIST

```

```

      DATA IERR /3*(-0),0,2*(-0)/

```

```

      CALL SYSTEMC (34,IERR)
      CALL SYSTEMC (119,IERR)
      WRITE (4,200)

```

```

      READ INPUT
      INITIALIZE VARIABLES

```

```

5 READ (5,100) N,K,SIGMA,DELT1,DELT2
  IF (K.LT.3 .OR. K.GT.N) GO TO 15
  IF (N .GT. 30) WRITE (6,500) N
  IF (DELT1 .GT. 10.*SIGMA) DELT1=10.*SIGMA
  IF (DELT2 .GT. 10.*SIGMA) DELT2=10.*SIGMA
  RES1=0.
  RES2=0.
  PI=3.1415926536
  NN=N/5+6

```

```

      IF (N .EQ. 50) NN=12
      ERROR=10.**(-NN)
      A=-10.*SIGMA
      B=+10.*SIGMA

C
C      EVALUATE TRIPLE INTEGRAL.
C
      IND=1
      RES1=SQUANK (A,B,ERROR,RUM,DIST)
      IF (N .EQ. K) GO TO 7
      IND=2
      RES2=SQUANK (A,B,ERROR,RUM,DIST)

C
C      DETERMINE FACTORIALS
C
      7 CONTINUE
      NFACT=1
      K2FACT=1
      K3FACT=1
      NKFACT=1
      NK1FACT=1
      DO 10 I=1,N
      NFACT=NFACT*I
      IF (I .LE. (K-2)) K2FACT=K2FACT*I
      IF (I .LE. (K-3)) K3FACT=K3FACT*I
      IF (I .LE. (N-K)) NKFACT=NKFACT*I
      IF (I .LE. (N-K-1)) NK1FACT=NK1FACT*I
      10 CONTINUE

C
C      COMPUTE AND WRITE PROBABILITIES
C
      PROB1=NFACT/(NKFACT*K3FACT)*RES1
      PROB2=NFACT/(NK1FACT*K2FACT)*RES2
      PROB=PROB1+PROB2
      WRITE (6,300) N,K,SIGMA,DELT1,DELT2,PROB1,PROB2,PROB
      GO TO 5
      15 WRITE (6,400) N,K
      STOP

C
      100 FORMAT (2I5,3F20.7)
      200 FORMAT (1H1//)
      300 FORMAT (1H ,4HN = .I2,3X,4HK = .I2,3X,8HSIGMA = .F10.7,3X,
      * 8HDELT1 = .F15.8,3X,8HDELT2 = .F15.8,3X/1H0,
      * 37HPROBABILITY (1 THROUGH K FUNCTION) = .F10.6/1H ,
      * 39HPROBABILITY (2 THROUGH K+1 FUNCTION) = .F8.6/1H ,
      * 22HPROBABILITY (TOTAL) = .F25.6/////)
      400 FORMAT (1H ,35H**** INVALID VALUE OF N OR K ****,3X,
      * 4HN = .I2,3X,4HK = .I2)
      500 FORMAT (21H ***** WARNING - N = .I2,
      * 51H, CHECK THE TOLERANCE LIMITS OF THE INTEGRALS ****//)
      END

C
C
C

```

CCCCC

66

•

20

FUNCTION FXY (V,U)

C
C
C

THIS ROUTINE PROVIDES THE INTEGRAND FOR THE FIRST INTEGRAL.

COMMON /COM1/ N,K,SIGMA,DEL1,DEL2

COMMON /COM2/ PI,W

COMMON /COM4/ IND

C

PHIUVW=1./((SQRT(2.*PI)*SIGMA)*EXP(-1.*(U-V+W)**2/(2.*SIGMA**2)))

NORMAL

CAPVW=FND ((V+W)/SIGMA)

NORMAL

CAPUVW=FND ((U-V+W)/SIGMA)

NORMAL

IF (IND.EQ. 2) GO TO 15

C

IF (N.EQ. 3) GO TO 2

IF (N.EQ. K) GO TO 5

IF (K.EQ. 3) GO TO 10

FXY=(CAPUVW-CAPVW)**(K-3)*PHIUVW*(1.-CAPUVW)**(N-K)

GO TO 25

2 FXY=PHIUVW

GO TO 25

5 FXY=(CAPUVW-CAPVW)**(K-3)*PHIUVW

GO TO 25

10 FXY=PHIUVW*(1.-CAPUVW)**(N-K)

GO TO 25

C

15 CONTINUE

C

IF (N.EQ. (K+1)) GO TO 20

FXY=(CAPUVW-CAPVW)**(K-2)*PHIUVW*(1.-CAPUVW)**(N-K-1)

GO TO 25

20 FXY=(CAPUVW-CAPVW)**(K-2)*PHIUVW

C

25 RETURN

END

C
C
C

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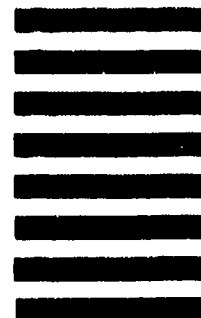


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